1306. *Proposed by Raymond Mortini, Universit é du Luxembourg, Esch-sur-Alzette, Luxembourg and Rudolf Rupp, Technische Hochschule Nürnberg, Nürnberg, Germany.*

Consider for $x \in (0, 1]$ and for $n \in \mathbb{N} := \{0, 1, 2, \ldots\}$ the equation

$$\frac{(1-x)^{n-1}}{x^{2n-1}} = 2. \tag{$*$}$$

Prove the following:

(1) For each n there exists a unique $x_n \in (0, 1)$ solving equation ($*$).
(2) Prove that $L := \lim_{n\to\infty} x_n$ exists.
(3) Determine L.

1307. *Proposed by Himadri Lal Das, Department of Mathematics, Indian Institute of Technology, Kharagpur, India.*

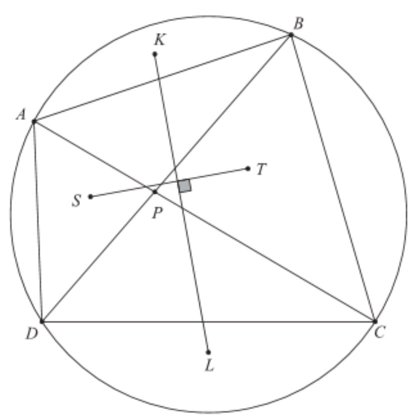
Let $\left[{n \atop k}\right]$ denote the number of permutations of n elements whose decomposition has k cycles and $\binom{n}{k}$ denote the number of k-combinations of n elements. Prove

$$\frac{1}{(p-1)!}\sum_{j=0}^{p}\sum_{k=0}^{n} k^j \left((n+1)\left[{p \atop j+1}\right] - \left[{p \atop j}\right]\right) = \binom{n+p+1}{n}$$

provided $0^0 = 1$ and n and p are positive integers.

1308. *Proposed by Tran Quang Hung, High School for Gifted Students, Vietnam National University, Hanoi, Vietnam.*

Let $ABCD$ be a cyclic quadrilateral. Let P be the intersection of two diagonals AC and BD. Let K and L be the circumcenters of triangles PCD and PAB, respectively. Let S and T be the symmedian points of triangles PAD and PBC, respectively. Prove that the two lines KL and ST are perpendicular. (The symmedian point of a triangle is the intersection of the three symmedian lines: these lines are the reflections of the medians in the corresponding angle bisectors).



1309. *Proposed by Ovidiu Furdui and Alina Sîntămărian, Technical University of Cluj-Napoca, Cluj-Napoca, Romania.*

Calculate the following expressions and prove your solutions are correct.

(a) $L = \lim_{n\to\infty} \int_0^1 \left(\frac{n}{n+\sqrt[n]{x}}\right)^n dx$

(b) $\lim_{n\to\infty} n\left(\int_0^1 \left(\frac{n}{n+\sqrt[n]{x}}\right)^n dx - L\right)$

1310. *Proposed by Eugen J. Ionaşcu, Columbus State University, Columbus, GA.*

Prove, for each $n \in \mathbb{N}$, that

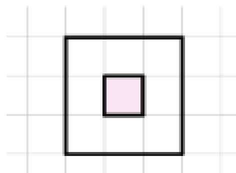
$$\sum_{k=0}^{n} \frac{(-1)^k \binom{n}{k}}{2k+1} > \frac{1}{2n+1}.$$

Problems from Crux MathemAttic, due March 15, 2026 (soft deadline)

MA351. The pairwise distances between nine cities are distinct. From each city, a traveller departs to visit the nearest city. Prove that some city is visited by at least two travellers.

MA352. A circle is divided by 27 points into 27 equal arcs. Each point is black or white. No two black points are adjacent or separated by one white point. Prove that there are 3 white points that are the vertices of an equilateral triangle.

MA353. Min and Max each have a 4×4 grid of 16 unit squares. Each of them removes three of the unit squares in their grid, and then computes the perimeter of their resulting shape. What is the maximum possible difference in their answers? Note: The perimeter of a shape is the sum of lengths of all the line segments that border the shape. For example, the following 3×3 square with the middle 1×1 square missing has the perimeter 16.



MA354. Show that, given five points in the plane in general position (that is, no three points are collinear), the number of convex quadrilaterals formed by these points is odd.

MA355.

(a) How many ways are there to pair up the elements of $1, 2, \ldots, 14$ into seven pairs so that each pair has sum at least 15?

(b) How many ways are there to pair up the elements of $1, 2, \ldots, 14$ into seven pairs so that each pair has sum at least 13?

(c) How many ways are there to pair up the elements of $1, 2, \ldots, 2024$ into 1012 pairs so that each pair has sum at least 2001?

Problems from Crux Olympiad Corner, due March 15, 2026 (soft deadline)

OC761. At a school, there are a number of clubs. A club is a set of students. Each club contains at least one student. A student may be in more than one club, but cannot be in every club. Surprisingly, for any two clubs A and B at the school, their union $A \cup B$ is also a club. Is it guaranteed that there is a club containing an even number of students?

OC762. Consider a 2024×2024 grid of unit squares. Two distinct unit squares are *adjacent* if they share a common side. Each unit square is to be colored either black or white. Such a colouring is called *evenish* if every unit square in the grid is adjacent to an even number of black unit squares. Determine the number of evenish colorings.

OC763. In a school, there are 1000 students in each year level, from Year 1 to Year 12. The school has 12000 lockers, numbered from 1 to 12000. The school principal requests that each student is assigned their own locker, so that the following condition is satisfied:

> For every pair of students in the same year level, the difference between their locker numbers must be divisible by their year-level number.

Can the principal's request be satisfied?

OC764. The set of natural numbers from 1 to 1000 inclusive is partitioned into two groups A and B of 500 numbers each. For an integer k, let N_k be the number of pairs (a, b) of a number $a \in A$ and a number $b \in B$ such that $a - b = k$. Prove that:

(a) in any such partition there exists k such that $N_k \geq 126$;

(b) there exists a partition such that $N_k \leq 250$ for every k.

OC765. Given a family $\mathcal{F}$ of 4-element subsets (4-tuples) of a given set of 5^m elements, where m is a fixed natural number. It is known that the intersection of no two 4-tuples in $\mathcal{F}$ consists of exactly two elements. Find the maximum possible value for the number of 4-tuples in $\mathcal{F}$.

Problems from Crux Mathematicorum, due March 15, 2026 (soft deadline)

5101. *Proposed by Mihaela Berindeanu.*

Let ABC be an acute triangle with the orthocenter H and the incenter I. Let H_a, H_b and H_c be the orthocenters of ΔIBC, ΔIAC and ΔIAB, respectively, and let

$$HA + HB + HC = IH_a + IH_b + IH_c.$$

Show that ΔABC is an equilateral triangle.

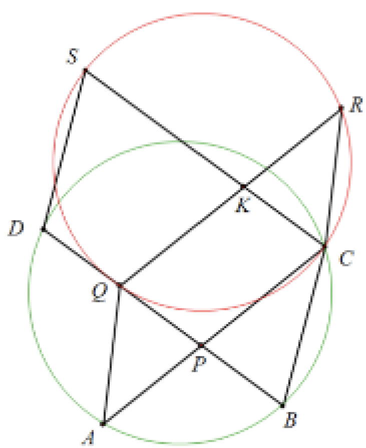
5102. *Proposed by Michel Bataille.*

Let n be a positive integer. Evaluate

$$\sum_{k=1}^{n} \binom{n+k}{2k} (-1)^{k-1} k.$$

5103. *Proposed by Xicheng Peng.*

As shown in the figure, diagonals AC and BD of quadrilateral $ABCD$ intersect at point P. Let Q be a point on line BD (not coinciding with B) such that $PQ = PB$. Construct parallelograms $CAQR$ and $DBCS$. Prove that A, B, C, D are concyclic if and only if Q, C, R, S are concyclic.



5104. *Proposed by Vasile Cirtoaje.*

Prove that 8 is the smallest positive value of k such that

$$\frac{a^2+b^2+c^2+d^2}{4} \le \left(\frac{a+b+c+d}{4}\right)^k$$

for all nonnegative real numbers a, b, c, d with $a \ge b \ge c \ge d$ and $(a+b)(c+d) = 4$.

5105. *Proposed by Tatsunori Irie.*

Find all functions $f : \mathbb{N}_0 \to \mathbb{N}_0$ such that for every $m, n \in \mathbb{N}_0$,

$$f(m^2 + mn + n^2) = f(m)^2 + f(m)f(n) + f(n)^2.$$

5106. *Proposed by Minh Ha Nguyen, modified by the Editorial Board.*

Let ABC be a scalene triangle and let (I) be its incircle. A point M varies on (I). Denote by H, K, L the perpendicular projections of M onto BC, CA, AB, respectively. Find the position of M such that the expression $MH + MK + ML$ attains its maximum (or minimum) value.

5107. *Proposed by Nikolai Osipov and Alex Chen.*

The base-b repunits are defined as

$$R_d(b) = 1 + b + \ldots + b^{d-1} = \frac{b^d - 1}{b - 1}.$$

For positive integers m, k, d, a such that $d \geqslant 2$, $a \geqslant 2$, prove that $R_d(a^m)$ is divisible by $R_d(a^k)$ if and only if m is divisible by k and $\gcd(m/k, d) = 1$.

5108. *Proposed by Huseyin Yigit Emekci.*

Let a, b, c, d be positive real numbers such that $a+b+c+d = 16$. Find the smallest value of

$$T = \frac{a}{b^3 + 32} + \frac{b}{c^3 + 32} + \frac{c}{d^3 + 32} + \frac{d}{a^3 + 32}$$

5109. *Proposed by Ion Patrascu.*

Let ABC be an isosceles right-angled triangle with $BA = BC$. On the segment BC we consider the point M to be mobile and denote by N its symmetry with respect to C. The points P and Q are the projections of B onto AM and AN respectively. Prove that the line PQ passes through a fixed point.

5110. *Proposed by Paul Bracken.*

Define the sequence of integrals I_n for $n \in \mathbb{N}$ as

$$I_n = \int_0^{\pi/2} (\sin^n x + \cos^n x)^{1/n} dx$$

and let $\Lambda = \lim_{n\to\infty} I_n$. Determine the limit

$$\lim_{n\to\infty} n^2(I_n - \Lambda).$$

Problems from Pi Mu Epsilon, due March 20, 2026 (soft deadline)

#1426: *Proposed by Robert Laniewski, Cape Town, South Africa.* Fermat's Last Theorem states that there are no integer solutions $a^n + b^n = c^n$ where a, b, and c are positive integers, and n is any integer greater than 2. In 2015, and then revised in 2020, a paper posted on arXiv[1] observed that there are no solutions to the Fermat equation where some of the components are prime. Specifically, there are no integer solutions where $a + b$, b, or c are prime, assuming that $c > b > a$, and $n > 2$ is any odd integer. Independently, we have found a one-page proof demonstrating that there are no integer solutions to the Fermat equation where a and b are both prime, and $n > 2$ is any odd integer (with no specific stipulation that $b > a$). Thus, prove elementarily that there are no solutions with a and b both non-zero positive prime numbers.

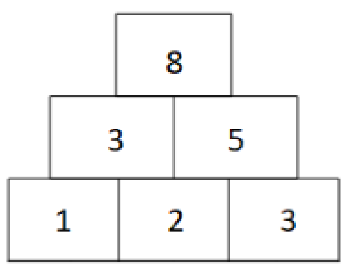
#1427: *Proposed Cameron, Elizabeth, Kayla and Steven Miller.* While admiring the numerous bridges in Venice on a water taxi, the four of us passed the time discussing another type of bridge, the card game, which led to the following interesting problem. *What is the fewest number of points you and your partner can have to make 7 spades under the following conditions: (i) you may distribute the cards however you want, but (ii) you must assume intelligent play by the opponents.*

#1428: *Proposed by Hongwei Chen, Christopher Newport University.* In the latest edition of Gradshteyn and Ryzhik's *Table of Integrals, Series, and Products* (8th ed., 2007), Entry 3.688.14 states that

$$\int_0^{\pi/2} \frac{1}{(\tan^\mu x + \cot^\mu x)^\nu} \frac{dx}{\tan x} = \frac{\sqrt{\pi}}{2^{2\nu+1}\mu} \frac{\Gamma(\nu)}{\Gamma(\nu + 1/2)} \quad (\nu > 0). \qquad (1).$$

A few tests for parameters μ and ν by Mathematica indicate (1) is incorrect. Can you offer a correction to (1)?

#1429: *Harold Reiter (UNC Charlotte) and Irina Xie (University of New Brunswick).* Here's the problem. An arithmetic triangle is a triangular array of positive integers such that each number on a row above the bottom row is the sum of the two integers directly below it. For example,



is an arithmetic triangle of size 3. Using the collection of 21 digits

$$\{0, 1, 1, 1, 1, 2, 2, 2, 3, 3, 3, 3, 4, 5, 5, 6, 6, 7, 7, 8, 9\},$$

build 15 different numbers so that each number in the bottom row is the sum of the two numbers above it. The digits in the diagram below are part of the 21 digits. Complete the triangle. Prove that your solution is unique.

#1430 (Reverse Tic-Tac-Toe): *Cameron Miller (TBD) and Steven J. Miller (Williams College).* In standard Tic-Tac-Toe, whomever gets three in a row (vertically, horizontallY or diagonally) wins. A simple calculation (though tedious if you don't use symmetry to combine cases) shows that if both play optimally, the game will always end in a tie. Consider now *Reverse Tic-Tac-Toe*, where the object is to force the other person to get three in a row. Does one of the players have a winning strategy (and if so who and what is it), or will the game always end in a tie if each play optimally?

Problems from the American Mathematical Monthly, due March 31, 2026

12559. *Proposed by Hideyuki Ohtsuka, Saitama, Japan, and Roberto Tauraso, Tor Vergata University of Rome, Rome, Italy.* Let t_n be the Thue–Morse binary sequence, defined by $t_{2n} = t_n$ and $t_{2n+1} = 1 - t_n$, with $t_0 = 0$. Evaluate $\sum_{n=1}^{\infty}(-1)^n\, t_n / n$.

12560. *Proposed by Wanlong Han (student), Xuchang University, Xuchang, China.* Let $f_1(x)$ be the inverse hyperbolic sine of x, and for $n \geq 2$, let $f_n(x) = f_1(f_{n-1}(x))$. For $b > 0$, evaluate $\lim_{n\to\infty} \sqrt{n}\, f_n(b/\sqrt{n})$.

12561. *Proposed by Donald E. Knuth, Stanford University, Stanford, CA.* How many undirected Hamiltonian cycles are there in the complete tripartite graph $K_{p,q,r}$ with nonzero part-sizes p, q, and r?

12562. *Proposed by Zachary Franco, Houston, TX.* A biased coin has probability p of landing heads up and probability $1 - p$ of landing tails up, where p is rational. Show that there are integers n and k such that out of n tosses, the probability that the number of heads is k or $k + 1$ equals the probability that it is $k + 2$.

12563. *Proposed by Tran Quang Hung, Hanoi, Vietnam.* Let P be the Fermat point of triangle ABC. (The point P minimizes the sum $AP + BP + CP$ of distances to the vertices.) Let Q be the point for which the vector equation $\overrightarrow{PQ} = \overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC}$ holds. Show that

$$PA^4 + PB^4 + PC^4 + PQ^4 = QA^4 + QB^4 + QC^4.$$

12564. *Proposed by Thomas Lam, NYU Courant Institute of Mathematical Sciences, New York, NY.* For a set $E \subseteq \mathbb{R}^2$, define the *pseudo-convex hull* of E to be the union of all segments of length at most 1 whose endpoints lie in E. Let $E_0 = \{(x, y) : xy = 0\}$, and, for $n \geq 1$, let E_n be the pseudo-convex hull of E_{n-1}. Is $\bigcup_{n=0}^{\infty} E_n$ equal to all of $\mathbb{R}^2$?

12565. *Proposed by Florin Stanescu, Cioculescu School, Gaesti, Romania.* Let A be an n-by-n complex matrix such that for any positive integer k, $\det\left(A^{2k} + 2A^k + I\right) = \det\left(A^{2k} + I\right)$, where I is the n-by-n identity matrix. Prove that A^n is the zero matrix.